# Resit
# First part (T1-T3) of
# Thermal Physics
# 2020-2021

**Saturday April 10, 2021**
**9.00-12.00**
**online**

PROBLEM 1

*Score: a+b+c+d+e=6+6+6+6+6=30*

A container with a volume of 1.0 $dm^3$ contains $2.0 \cdot 10^{23}$ He atoms. The temperature of the He gas is 54.4 K. You can assume that the He behaves like an ideal gas.

Given: $M_{He} = 4.0$ g $mol^{-1}$, $M_{N_2} = 28.0$ g $mol^{-1}$, $M_{O_2} = 32.0$ g $mol^{-1}$.

a) Determine the gas pressure.
b) What is the root mean square (RMS) speed of the molecules at this temperature?
c) What is the internal energy of the gas in the container?
d) Now you replace every He atom by a $N_2$ molecule. How do temperature, RMS speed and internal energy of the gas change? Explain.
e) Draw a sketch of the Maxwell distribution. Roughly approximate the fraction of $O_2$ molecules that have molecular speeds in the range between 1500 m/s and 1600 m/s when the temperature is 300 K.

PROBLEM 2

*Score: a+b+c+d +e=6+6+6+6+6=30*

Consider a hypothetical engine which connects two (infinite) reservoirs and undergoes the following *reversible* processes:

*i.* isothermal expansion at temperature $T_h$

*ii.* adiabatic expansion to temperature $T_c$

*iii.* isothermal compression at temperature $T_c$

*iv.* adiabatic compression to the initial state

In every cycle, a quantity of heat $Q_{in}$ flows from reservoir A into the engine and a quantity of heat $Q_{out}$ flows from the engine into the reservoir B.

a) Sketch the thermodynamic cycle in the usual $(p,V)$-diagram. Clearly indicate the direction of the cycle. In which steps does the heat flow and in which steps is the engine doing work?

b) Sketch the thermodynamic cycle in a (slightly less usual) $(T,S)$-diagram. Indicate where the entropy of the engine changes and how these changes relate to the entropies of reservoirs A and B.

c) Use entropy arguments to show, that the efficiency of the cycle equals $\eta = 1 - T_c/T_h$.

d) The coal power plant in Eemshaven produces about 12 TWh of electricity per year from about 3 million tons of coal (thermal energy content of coal: 9 kWh kg$^{-1}$). Determine the efficiency of the power plant and estimate $T_c$ and $T_h$ assuming the power plant is an idealized Carnot engine. Is the result for $T_h$ realistic? Estimate the efficiency of a primitive steam engine (use realistic numbers) and compare.

e) Until now we have assumed infinite reservoirs A and B. Now assume both reservoirs are finite (and have the same heat capacity). Discuss the effect on the thermodynamic cycle and on the reservoir temperatures.

PROBLEM 3

*Score: a+b+c =6+6+6=18*

The thermal diffusion equation for a sphere can be written as

$$\frac{\partial T}{\partial t} = D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$$

if there is no dependence on the angular coordinates. In this equation $r$ is the radial coordinate, $D$ is the diffusion coefficient and $T$ is the temperature.

a) Give a general solution for the steady state case.
b) Give the solution for the boundary condition of a spherical animal of radius $r_0$, with body temperature $T_{body}$ for $r = r_0$ and a temperature of the outside medium $T_{outside}$ for $r \to \infty$.
c) How much heat does the animal loose to the medium per second? The thermal conductivity of the medium is $\kappa$.

PROBLEM 4

*Score: a+b=6+6=12*

Typical turbomolecular pumps can generate a vacuum of about $1 \cdot 10^{-10}$ bar. Assume the pump to be at $T = 25° \, C$ and working on a gas primarily consisting of $N_2$ molecules.

a) Calculate the mean free path of the molecules and the collision frequency.

b) Do the molecules that pass the turbomolecular pump have the same Maxwell Boltzmann distribution of speeds that the gas in the vacuum chamber has? (Hint: assume that the pump has a diameter of 10 cm. Use the result for the mean free path to argue.) If the answer is no, what speed distribution do these molecules have?

Collision diameter $N_2$: 395 pm.

**Solutions**

PROBLEM 1

a)

$$pV = Nk_BT$$

$$p = \frac{Nk_BT}{V} = \frac{2 \times 10^{23} \times 1.38 \ \times 10^{-23}\,{}^{\mathrm{J}}\!/_{\mathrm{K}} \times 54.4\ \mathrm{K}}{10^{-3}\mathrm{m}} \approx 150\ \mathrm{kPa}$$

b)

$$v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{\frac{3k_BT}{m}}$$

$$m = \frac{0.004\ \mathrm{kg\ mol^{-1}}}{N_A} \approx 6.6 \times 10^{-27}\mathrm{kg}$$

$$v_{rms} = \sqrt{\frac{3 \times 1.38 \cdot 10^{-23}\,{}^{\mathrm{J}}\!/_{\mathrm{K}} \times 54.4\ \mathrm{K}}{6.6 \times 10^{-27}\mathrm{kg}}} \approx 584\ \mathrm{ms^{-1}}$$

c) The internal energy of a (monoatomic) ideal gas is sum of the particle kinetic energies.

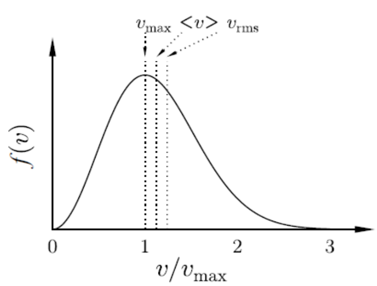
$$\langle u\rangle = \frac{1}{2}m\langle v^2\rangle$$

$$\langle U\rangle = N\langle u\rangle = N\frac{1}{2}m\langle v^2\rangle = N\frac{1}{2}m\frac{3k_BT}{m} = \frac{3}{2}Nk_BT \approx 225\mathrm{J}$$

d)

The temperature would not change, as the number of particles stays the same. The rms speed, however, does depend on $\frac{1}{\sqrt{m}}$. The mean kinetic energy is defined as $\langle E\rangle = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$ and does not depend on $m$, so neither does $U$.

e)




$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} v^2dv\, e^{-\frac{mv^2}{2k_BT}}$$

$$m = \frac{0.032 \text{ kg mol}^{-1}}{N_A} \approx 5.3 \times 10^{-26}\text{kg}$$

$$f(1550 \text{ ms}^{-1}) = \frac{4}{\sqrt{\pi}}\left(\frac{5.3 \cdot 10^{-26}\text{kg}}{2k_B 300 \text{ K}}\right)^{3/2} (1050 \text{ ms}^{-1})^2 dv\, e^{-\frac{5.3 \cdot 10^{-26}\text{kg}\left(1050 \text{ ms}^{-1}\right)^2}{2k_B 300\text{K}}}$$

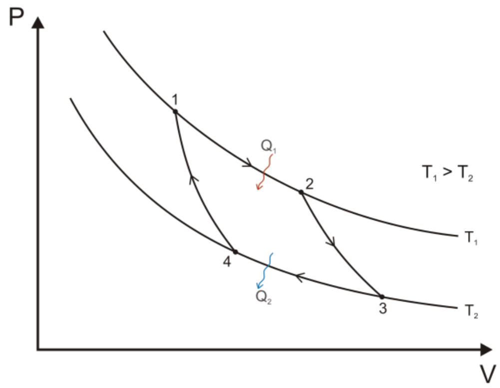
$$\approx 1.2 \cdot 10^{-4} \text{ sm}^{-1} dv$$

$$\Delta v = (1500 - 1600)\text{ms}^{-1} = 100 \text{ ms}^{-1}$$

The fraction is approximately 0.012%.

PROBLEM 2

a)



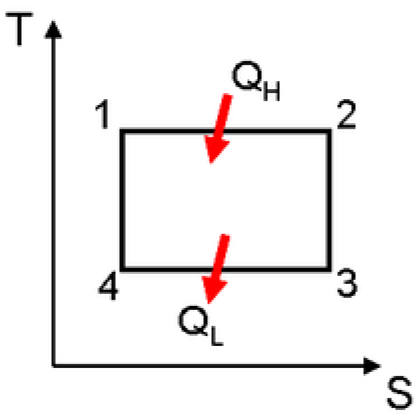

Heat into the engine: i)

Heat out of engine: iii)

Work by engine: i), ii)

Work on engine: iii) iv)

b)




Entropy changes in the isothermal steps i) and iii) (heat flows to keep $T$ constant). Adiabatic means no heat flow, i.e. no entropy change.

$$\Delta S_A = \int \frac{dQ}{T_h} = \frac{\Delta Q}{T_h} = \frac{Q_h}{T_h}$$

$$\Delta S_B = \frac{Q_c}{T_c}$$

c) $$\Delta S_h = \Delta S_c$$

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

$$\eta = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$$

d)

$$\eta = \frac{12\ \text{TWh}}{3 \cdot 10^9 \text{kg} \ \times 9\ \text{kWh kg}^{-1}} \approx 0.44$$

The cooling water is liquid water from outside the plant (e.g. the sea), let's say $T_c = 300$ K.

$$\eta = 1 - \frac{T_c}{T_h}$$

$$T_h \approx \frac{T_c}{0.56} = 535\ \text{K}$$

This is for an idealized Carnot process. The actual temperatures are much higher!

Steam engine: $T_h$=100°C=373K, $T_c$=50 °C=323K, $\eta = 1 - \frac{T_c}{T_h} \approx 0.1$

e) The reservoirs will converge to equal temperatures. At the same time, the efficiency will gradually drop zo zero.

PROBLEM 3

a)

$$D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

$$r^2\frac{\partial T}{\partial r} = const$$

$$T(r) = A + \frac{B}{r}$$

b)

$$T(r_0) = T_{\text{body}} = A + \frac{B}{r_0}$$

$$T(r \to \infty) = T_{\text{outside}}$$

$$A = T_{\text{outside}}$$

$$B = \left(T_{\text{body}} - T_{\text{outside}}\right)r_0$$

c)

$$J_r = -\kappa\frac{\partial T}{\partial r} = \kappa r_0\left(T_{\text{body}} - T_{\text{outside}}\right)\frac{1}{r^2}$$

$$J_{r_0} = \kappa r_0\left(T_{\text{body}} - T_{\text{outside}}\right)\frac{1}{{r_0}^2}$$

$J_{r_0} 4\pi {r_0}^2 = 4\pi\kappa r_0\left(T_{\text{body}} - T_{\text{outside}}\right)\frac{1}{{r_0}^2}$ per second

PROBLEM 4

a)

$$\lambda = \frac{1}{\sqrt{2}n\sigma}$$

$$n = \frac{N}{V} = \frac{p}{k_B T} = \frac{1 \times 10^{-5}\text{Pa}}{1.38 \times \frac{10^{-23}\text{J}}{\text{K}} \times 298\text{ K}} \approx 2.4 \times 10^{15}\text{m}^{-3}$$

$$\sigma = \pi \times (395 \times 10^{-12})^2 = 4.9 \times 10^{-19}\text{m}^2$$

$$\lambda = \frac{1}{\sqrt{2} \times 2.4 \times 10^{15}\text{m}^{-3} \times 4.9 \times 10^{-19}\text{m}^2} = 601\text{ m}$$

This is huge, because of the low pressure, molecules essentially do not collide with each other in a normal-sized container.

b) Despite the macroscopic diameter of the pump, we are dealing with effusion rather than with a regular Maxwell-Boltzmann distribution, simply because the mean free path is large as compared to the diameter of the opening.